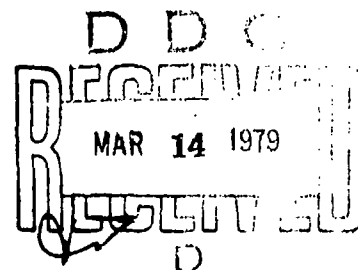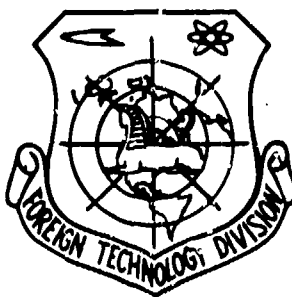# FOREIGN TECHNOLOGY DIVISION

INTERIOR BALLISTICS OF BARREL SYSTEMS AND
SOLID-PROPELLANT ROCKETS
(Chapter XII)

by

M. Ye. Serebryakov

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# EDITED TRANSLATION

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SOLID-PROPELLANT ROCKETS (Chapter XII)

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# U. S. BOARD ON GEOGRAPHIC NAMES TRANSLITERATION SYSTEM

| Block | Italic | Transliteration | Block | Italic | Transliteration |
|-------|--------|-----------------|-------|--------|-----------------|
| А а | *А а* | A, a | Р р | *Р р* | R, r |
| Б б | *Б б* | B, b | С с | *С с* | S, s |
| В в | *В в* | V, v | Т т | *Т т* | T, t |
| Г г | *Г г* | G, g | У у | *У у* | U, u |
| Д д | *Д д* | D, d | Ф ф | *Ф ф* | F, f |
| Е е | *Е е* | Ye, ye; E, e* | Х х | *Х х* | Kh, kh |
| Ж ж | *Ж ж* | Zh, zh | Ц ц | *Ц ц* | Ts, ts |
| З з | *З з* | Z, z | Ч ч | *Ч ч* | Ch, ch |
| И и | *И и* | I, i | Ш ш | *Ш ш* | Sh, sh |
| Й й | *Й й* | Y, y | Щ щ | *Щ щ* | Shch, shch |
| К к | *К к* | K, k | Ъ ъ | *Ъ ъ* | " |
| Л л | *Л л* | L, l | Ы ы | *Ы ы* | Y, y |
| М м | *М м* | M, m | Ь ь | *Ь ь* | ' |
| Н н | *Н н* | N, n | Э э | *Э э* | E, e |
| О о | *О о* | O, o | Ю ю | *Ю ю* | Yu, yu |
| П п | *П п* | P, p | Я я | *Я я* | Ya, ya |

*ye initially, after vowels, and after ъ, ь; e elsewhere.
When written as ё in Russian, transliterate as yё or ё.

## RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

| Russian | English | Russian | English | Russian | English |
|---------|---------|---------|---------|---------|---------|
| sin | sin | sh | sinh | arc sh | $\sinh^{-1}$ |
| cos | cos | ch | cosh | arc ch | $\cosh^{-1}$ |
| tg | tan | th | tanh | arc th | $\tanh^{-1}$ |
| ctg | cot | cth | coth | arc cth | $\coth^{-1}$ |
| sec | sec | sch | sech | arc sch | $\operatorname{sech}^{-1}$ |
| cosec | csc | csch | csch | arc csch | $\operatorname{csch}^{-1}$ |

| Russian | English |
|---------|---------|
| rot | curl |
| lg | log |

## Chapter XII
### Bore Systems with ~~Emission~~ _Escape_ of Gases during ~~Combustion~~ _Burning_ of Powder

Depending on the design and construction of the system gases can flow either from a chamber of constant volume with a nozzle or from the bore of the gun; gases can flow through small or large _vents_ of varying _shape_ (one or several round _vents_, ~~parallel to~~ _perpendicular_ or at angles to the axis of the system, narrow circular aperture).

Depending on the shape, _vents_ will vary in size and in the gas flow coefficient characterizing the compression of the flow.

We list the following as examples:

1) the flow proceeds from a chamber of constant volume with a nozzle in a gas _operated_ gun;

2) the flow proceeds from a chamber of a recoilless gun in the direction opposite to the motion of the projectile concurrent with an increase in the volume of the bore resulting from the movement of the projectile along the bore of the barrel;

3) the flow proceeds from the bore of a barrel of mortar through a narrow circular space between the mine and the smooth bore of the barrel in direction of the motion of the mine; in this case some of the gas overtakes the mine and bursts forward.

The firing in the second case appears the most complex. In all cases at high pressures the law of the burning rate is expressed by the relation $\varkappa = u_1 P$.

### 12.1. Concept on the Characteristics of Interior Ballistics of Gas _operated_ Guns

All equations derived in Chapter III for the combustion of powder in a bomb with _a vent_ at high pressures are also applicable to separated combustion chambers in gas _operated_ guns. The foundations of theories for these guns and the order of their

construction was developed by V.M. Trofimov in 1923-1925.

The diagram for such a gun is presented in Fig. 12.1.
Powder gases in a separate combustion chamber flow through a
nozzle into the bore of the barrel; the projectile is some distance
$l_{0\ bore}$ from the anterior section of the nozzle (a type of fore-
chamber). The highest pressure in the combustion chamber reaches
2000-2500 $kg/cm^2$; the presence of the nozzle between the chamber
and the bottom of the projectile enables the regulation of the gradual
flow of gas into the barrel bore and the obtention of a low and
almost constant pressure. This in turn permits the use of projectile
with thinner walls, an increase in the charge coefficient for the
explosive, and a significant decrease in the thickness of the
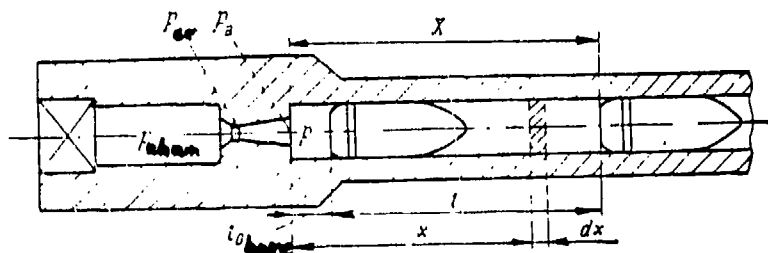bore walls, which also decreases the weight of the barre .



Fig. 12.1 Diagram for Gas Dynamic Gun

Powder combustion and the laws of pressure change of powder gases
will be essentially identical under the same charge conditions $(W_o,$
$F_{min}, \omega, \Delta, I_f)$ in a bomb with a nozzle, described in Chapter III,
and in a chamber of a gas operated gun. However, in experiments
with a bomb with a nozzle, gases flow through the nozzle into a
space with a constant atmospheric pressure; during combustion of
the charge in a chamber of a gas operated gun, the gases flow through
the nozzle into a closed space which gradually increases with the
motion of the projectile. In this post-projectile space gas pres-
sure is close to constant. This pressure appears as counterpres-
sure with respect to the pressure within the chamber. Usually,
during combustion of the powder charge the pressure within the
barrel bore is several times less than in the chamber, so that it
will not affect powder combustion and the supercritical nature of

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gas flow. After combustion of the charge the pressure within the combustion chamber only falls, but there can be a moment when it is equal to the pressure within the bore. Prior to this moment, the flow regime is already subcritical and the flow pattern depends on the counterpressure.

As a basis for the problem of interior ballistics for gas-operated gun, V.M. Trofimov proposed the following assumptions:

1. Processes occuring within the barrel bore do not affect powder *burning* within the chamber and gas flow in the nozzle $(P_{bore} << P_{cham})$.

2. *Burning.* of powder follows the geometric law of *burning*

3. Powder has a constant *burning* surface.

4. The law of the burning rate is given by the formula $u = u_1 p$.

5. The pressure within the chamber is highest at the end of combustion $p_{max} = p_f$

6. For the first period after firing, the process in the chamber is isothermal and the flow through the nozzle adiabatic. However, the gas temperature within the chamber $T_{cham}$ is less than the combustion temperature $T_1$.

Subsequent research indicated that $T_{cham\ a}$ $/T_1 = \tau_{cham\ a}$ was close to 0.90.

7. The pressure at the bottom of the projectile results from shock waves formed in the gas flow during its *escape* from the nozzle and propagated in the bore with the speed of sound in *stationary* ~~still~~ gas.

Later (in 1950) the following were assumed:

1.The composition of the combustion products is constant; $c_p$ and $c_w$ are average and constant during powder *burning*.

2. Heat emission is evidently not considered; however, this can be accomplished by conventional procedure by decreasing the

force of the powder f or increasing the index $\theta$-k-1.

3. Gas in the chamber is motionless; consequently, the pressure p, gas temperature T, the specific volume w and the gas density $\rho$ are identical in a given moment in the entire chamber.

Hence, the general relation follows:

$$p(w-a) = RT_{cham}$$

4. The gas motion within the nozzle is steady and one dimensional, so that for each cross section at a distance x from the entrance cross section, this equality is correct:

$$p_x w_x = RT_x.$$

The assumptions presented above for deriving theoretical functions and the functions themselves adequately reflect the processes occuring during powder *burning* in a chamber with a nozzle.

The pressure within the chamber reaches a maximum at the end of *burning* not only for powder with a constant *burning* surface but also for slightly regressive powder (ribbon); for powder with seven *perforations* $p_{max}$ is reached at the moment of disintegration $\psi_s$ when $\sigma_s$ has the highest value $\sim 1.37$.

Gas temperature in the chamber during the initiation of burning decreases to $T_{cham\ av} = 0.92\ T_1$ and thereafter remains fairly constant to the end of powder burning or the disintegration.

The theoretical relations used are applicable to gas-operated guns when the pressure within the bore $p_{bore}$ is less than the critical pressure $p_{cr}$ with respect to the pressure within the chamber.

During firing from a gas-operated gun the division into periods is somewhat different than for conventional guns.

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1. The period before the cutting of the *driving* band in the groove (typical).

2. The second period *lasts* up to the end of powder burning and the emission of gases into the barrel bore; gas pressure in the chamber significantly exceeds gas pressure in the bore which increases gradually.

3. The first phase of the second period *extends* from the end of powder burning to the moment of pressure equalization in the chamber and in the bore; at this time, the pressure at the bottom of the *projectile* $p_{pr}$ is greater than the average pressure in the bore under the impact of the gas on the *projectile* bottom (the reverse of that which is observed with conventional guns).

4. The second phase of the second period *is the* period of adiabatic expansion of gases to the ejection of the *projectile* from the barrel bore.

5. Period of the aftereffect of the gas (in the conventional sense).


Some Relations for Ballistic Elements


The force acting in the bore of the gas-operated gun on the bottom of the *projectile* has the rate v (according to V.M. Trofimov).

$$\Pi = s\rho_{bore}\, a_0\, (u_a - v),\qquad (12.1)$$

where $u_a$ = rate of gase flowing from the nozzle;

$a_0$ = speed of sound of the escaping gas

$\rho_{bore}$ = mass density of the gases in the bore:

$$\rho_{bore} = \frac{\omega\eta}{gs(l_{0\,bore}+l)};$$

$\omega\eta$ = weight of gas escaping from the chamber into the bore.


It follows that

$$\Pi = \frac{\omega}{g}\,\eta_\kappa\,\frac{a_0(U_a-v)}{(l_{0\,bore}+l)}\,\frac{p_{cham}-p_s}{p_{cham}\,max-p_s},\qquad (12.2)$$

where

$$\eta = \eta_\kappa\,\frac{p_{cham}-p_s}{p_{cham}\,max-p_s} = \eta_\kappa\,\frac{I}{I_\kappa}.$$

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After some conversions V.M. Trofimov gives the final expression for the force $\pi$:

$$\Pi = \frac{\omega}{l_{0_{cham}} + i}\frac{a_0^2}{g}\Phi(t), \qquad (12.3)$$

where $\phi(t) = \eta_K \dfrac{P_{cham} - P_\beta}{P_{cham\,max} - P_\beta}\left(1 + \dfrac{aP_{cham}}{f\sqrt{e}}\right)$ for the period of powder

burning; $\phi(t) = \left[1 - (1 - \eta_k) - \dfrac{P_{cham}}{P_{cham\,max}}\right]\left(1 + \dfrac{aP_{cham}}{f\sqrt{e}}\right)$ after powder burning.

The equation for the motion of the ~~projectile~~ is:

$$\frac{\varphi g}{g}\cdot\frac{dv}{dt} = \pi.$$

The formula for the pressure of gasses within the bore is:

$$p_{cham} = \frac{f\omega\eta_i\tau}{s(l_{0_{bore}} + i)}\cdot\frac{P_{cham} - p_\beta}{P_{cham\,max} - p_\beta}. \qquad (12.4)$$

The distribution of gases along the barrel bore is given by the equation:

$$P_{bore}x = P_{pr}\left\{1 + \frac{1}{2}\frac{\omega}{\varphi_1 q}\eta\left(1 - \frac{x^2}{X^2}\right) - \right.$$

$$\left. - \frac{1}{2}\frac{\omega}{g}\frac{\eta}{s}\left(1 - \frac{x^2}{X^2}\right)\left[\frac{U_a}{X}(U_a - v) - \frac{dU_u}{dt}\right]\right\}. \qquad (12.5)$$

The pressure $p_{pr}$ is greater than $p_{av}$ in the bore.



1) $P_{cham}$; $P_{bore} > V$
2) $P_{cham}$
3) $P_{max\,cham}$
4) $V_b$
5) $P_{bore}$
6) Grain disintegration
7) Phase I
8) Phase II

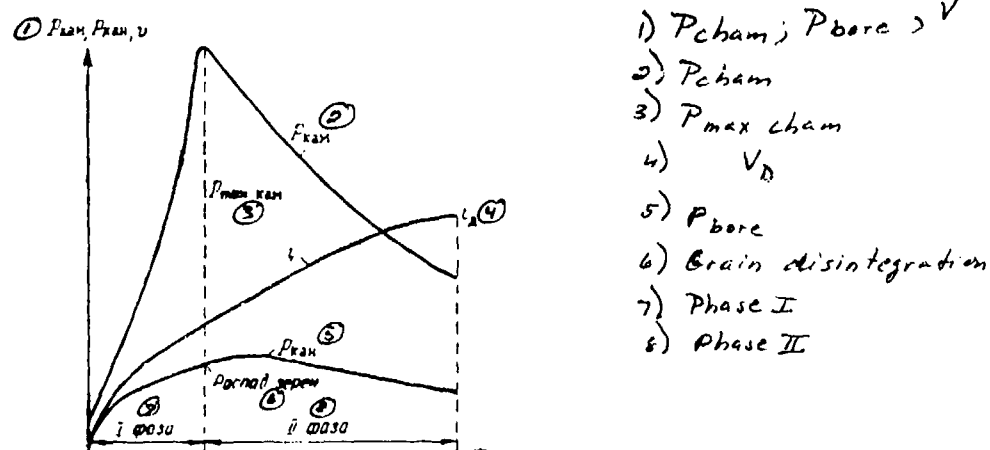Fig. 12.2. Curves p, l and v, l in the bore of a gas-operated gun.

Fig. 12.2 illustrates the curves of the pressure within the

chamber $p_{cham}$, the pressure within the bore $p_{bore}$, and the
velocity of the ~~projectile~~ v as a function of the trajectory of the ~~projectile~~
The graph shows that the pressure within the barrel bore increases
slowly and after reaching maximum pressure also decreases very
slowly; the curve $p_{bore}$, l is similar to the curve for constant
pressure; therefore, the curve for ~~projectory~~ veolocity v, l
resembles a straight line for a substantial section of the tra-
jectory (under the effect of almost constant pressure the inc-
rease in velocity is also constant). The curve of the chamber
pressure at corresponding moments varies withing much greater
limits.

12.2. Characteristics     of Interiror Ballistics for a Recoilless
                          Gun (RG)

The lack of recoil or total balance of the barrel gun is
achieved by the action of the force of the reaction of a fraction
of the poder gases escaping from the chamber through the nozzle
at the bottom of the ~~sleeve~~ or breech in a direction opposite to
the motion of the ~~projectile~~ This also relieves the load on the gun
carriage which exists during recoil with the gun carriage becoming
only a directional mount supporting the barrel. This pronounced-
ly decreases the weight of the system as a whole and significantly
increases the metal use coefficient $\eta_b = E_b/Q_b = m v_b^2/2 Q_b$
to $130-150$                             in place of the conventional 70-130.

However, this is attained primarily because of the signi-
ficant increase in the weight of the charge and the volume of the
chamber for a given initial velocity of the ~~projectile~~ $v_d$ and at the
highest pressure of the propellant gases $p_{max}$. Actually, in addi-
tion to the amount of gases necessary to obtain the given ballistics
in the given barrel (active part of the charge), a certain weight
of charge is required to form gases which a re propelled in the
reverse direction through the nozzle and create the force of the
reaction to balance the barrel (passive part of the charge).

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As a rule, for the given $v_D$ and $p_{max}$, the weight of the charge and the volume of the chamber of the recoilless *guns* are 2 to 3 times greater than the weight of the charge and the volume of the chamber in conventional *guns*. Consequently, recoilless *guns* are used to obtain relatively low *projectile* velocities ($v_D$ =350-500 m/sec).

Because of the increase in the total charge weight, the output coefficient for the charge $\eta_\omega = \varepsilon_D /\omega = m v_D^2 /2\omega$ in recoilless *guns* is significantly less than for conventional *guns* $\eta_\omega$ =20-50 t·m/kg rather than 120-140.

In order that the process during the escape of gas through relatively large cross sections of the nozzle ($F_{cr}$: $s \approx 0.6$) should proceed relatively intensely, the surface area of the charge should be very large with the powder *curve* relatively thin, since it is known that $S_1/\Lambda_1 = x/e_1$.

Therefore, the end of burning occurs early and the curve p, l is *peaked*; $\eta_D$ =0.15-0.45 rather than 0.40-0.70 as for conventional *guns*. Recoilless *guns* have an essential tactical flaw: during firing they are decamouflaged by the gases escaping from the nozzle at an angle to the ground which lift up large puffs of dust and thereby disclose the location of the *gun*. These gases create the impact wave which increases the pressure around the *gun* and the *gun* team should be either to the side of the *gun* or at a sufficiently large distance from it.

Because of their relatively light weight, the recoilless *guns* are also used *as* direct support for the infantry. These *guns* were widely used by the *U.S.* Army in the Korean War. According to reports, the U.S.A. and England have 120 mm recoilless antitank weapon.

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## Types of Recoilless *Guns*

Weapons with open*-ended*, cylindrical *barrels* (Fig. 12.3).
This type of recoilless *gun* on a light weight tripod with
70 mm caliber was developed and tested in 1916 by the engineer
D. Ryabushinskii. In this case the weight of the barrel was
7 kg and the weight of the projectile 3 kg, weight of gunpowder
was 0.3000-0.4000 kg; $v_b = 60$ m/sec; a distance of 320 m was achieved
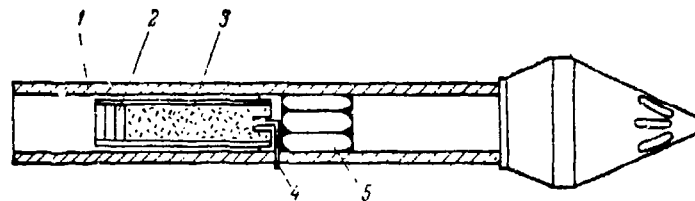during firing.



Fig. 12.3. Diagram for an open-ended barrel
1) barrel; 2) *base plate* 3) charge; 5) stabilizer.

The German Army in World War II used this type of weapon
with supercaliber charge known as the antitank weapon Faust-patrone;
later the U.S.A. used the "bazooka". Many countries use this
*design* for their grenade throwers with hollow-charge shells.
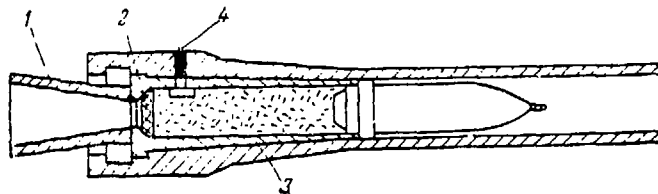


Fig. 12.4. Diagram for a central nozzle and *base plate*
1) nozzle; 2) *base plate* 3) *sleeve* 4) ignition

Recoilless *guns* with axial withdrawal of gases ha*ve* *vents*
at the bottom of the barrel sleeve and single-flow widening nozzle
(Fig. 12.4). ignit*ion occurs at the* side through a special channel.
To increase the initial pressure to ensure uniform burning of the
charge, the nozzle at the bottom of the barrel sleeve is closed
by *a base plate* (wooden or plastic) which is ejected at a
pressure close to the pressure for boosting the projectile $p_0$.

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The drawback of this type of design is that certain amounts of still unburned grains (fo) the powder charge are drawn in through the [opening large] of the nozzle by the gases; this causes variations in the firing.
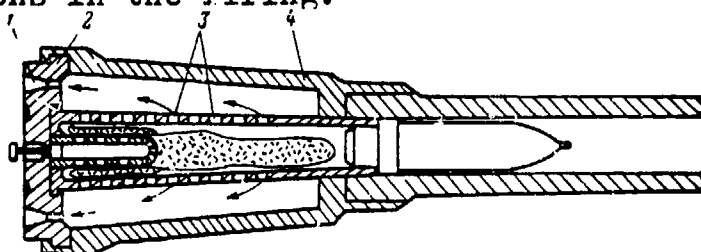


Fig. 12.5. Diagram for a perforated barrel sleeve
1) nozzle; 2) bolt; 3) perforations; 4) chamber.

Recoilless *guns* with a perforated barrel sleeve (Fig. 12.5)
*have* · lateral surfaces *with*

many small *vents* to hinder the ejection of powder.
After esc ping initially *into* the [side chamber exterior]
the gases subsequently flow through several nozzles at the bottom of the chamber. Ignition is of a conventional type.
This type of recoilless *gun* was used by the U.S. Army in the Korean (War).

Conditions for Balancing the System

Assume the recoilless *gun* is secured to the pins of the mount; the force of the reaction in the supports of the pins is designated as P (for each support P/2).(Fig. 12.6).



Fig. 12.6. Diagram of the forces acting on a recoilless
*gun*

Let's consider the moment when the projectile with the mass m has the velocity v; gases flowing under pressure into the

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chamber p through the critical cross section of the nozzle $F_{cr}$ expand and passing through the external section of the nozzle $F_a$, they have t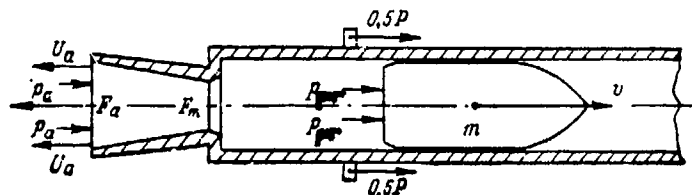he pressure $p_a$, which is usually significantly greater than atmospheric pressure ($p_a$ of the order 5-7 kg/cm$^2$); the velocity of gas flow in the outgoing cross section is $U_a$.

Let's use the theorem of mechanichs: the momentum of a particle of the system under the action of internal forces is equal to the sum of *momenta* of external forces.

1. The momentum under the effect of internal forces: projectile, +m dv; gases flowing through the nozzle $-\dfrac{G_{sec}}{g}\, dt\, U_a$.

2. External forces: *momentum* of the reaction of the support $\pm P dt$; *momentum* of the force of gas pressure in the external cross section of the nozzle is $p_a F_a\, dt$. We obtain

$$m\, dv - \frac{G_{sec}}{g}\, dt\, U_a = p_a F_a\, dt \pm P dt. \quad (12.6)$$

But $\dfrac{G_{sec}}{g}\, U_a + F_a p_a = R$ is the force of the reaction of gases on the barrel; $R = \varsigma\, F_{cr}\, p_1$

where $\varsigma = f\left(\dfrac{d_a}{d_{cr}}\right) = f_1\left(\dfrac{F_a}{F_{cr}}\right)$; this is presented in the table in chapter II (cf. page 100).

To calculate the loss, the *factor* is introduced $\chi < 1$:

$$\chi = \varphi_2 \varphi')$$

where $\varphi_2 \approx 0.95$, is the coefficent of gas expansion;
$\varphi' \approx 0.90$ is the velocity coefficient;
$\chi \approx 0.85$.

Then

$$R = \chi\, \varsigma\, F_{cr.}\, p. \qquad (12.7)$$

From the equation for the motion of the projectile $\varphi m \dfrac{dv}{dt} = sp$

we have $m\, dv = \dfrac{s}{\varphi}\, p\, dt.$ $\qquad (12.8)$

By substituting the expression (12.7) and (12.8) into (12.6)
and cancelling for dt, we obtain

$$\frac{s}{\varphi}\rho = \chi \zeta F_{cr} P \pm P. \qquad (12.9)$$

For a balanced (gun) P should equal zero. Then, the
conditions for equilibrium are written as follows (after cancelling
p):

$$\frac{s}{\varphi} = \zeta \chi F_{cr} )$$

from which we find the ratio $\frac{F_{cr}}{s}$, which is the critical cross
section of the nozzle divided by the cross section of the bore of
the gun to obtain a recoilless barrel:

$$\frac{F_{cr}}{s} = \frac{1}{\varphi \chi \zeta} . \qquad (12.10)$$

Since $\zeta = 1.6 - 1.7$ depends on the ratio $\frac{F_a}{F_{cr}}$; then $\frac{F_{cr}}{s}$
depends on $\frac{F_a}{F_{cr}}$, $\varphi$ and $\chi$ (loss factor).

In calculating the coefficient $\varphi$ for recoilless guns using
the formula $\varphi = \varphi_1 + \frac{1}{3}\frac{\omega_{dv}}{2}$ one must account for only that fraction
of the weight of the gas powder mixture which results in the
motion of the projectile. To calculate this, one must first de-
termine the fraction of gases and charge which escape through
the nozzle in the opposite direction.

At the end of the first period, i.e., at the end of the burning
of the powder, this fraction is calculated by the existing formula:

$$\eta_\kappa = \frac{\varphi_2 F_{cr} K_0 I_\kappa}{\omega \sqrt{f \tau_{av}}} .$$

where $\tau_{av}$ is determined using the formula of P.N. Shkvornikov:

$$\tau_{av} = \frac{1}{1 + \omega \eta_\kappa} )$$

where $\tau_{av}$ with the variable $\eta_\kappa$ can be 0.9; $\eta_\kappa = 0.5 - 0.6$ is typical for
recoilless guns.

After burning of the powder the escape of gases continues.
Since the pressure in recoilless guns in the second period
decreases rapidly and the muzzle velocity of the projectile is $v_b$
only slightly greater than $v_f$, then in the first approximation
at the moment of ejection of the projectile the fraction of gases
flowing through the nozzle from the chamber will be $\eta_b = \eta_\kappa \frac{v_b}{v_f}$ ;

with $v_\Delta/v_\varphi \approx 1.05 - 1.10$    $\eta_\Delta \approx 0.65 \approx \frac{2}{3}$.

In this case a fraction of the charge $1-\eta_\Delta$ will follow the projectile and we obtain the expression for the coefficient

$$\varphi = \varphi_1 + \frac{1}{3}\frac{\omega}{q}(1 - \eta_{\Delta\,av}).$$

With special efforts a more detailed calculation of secondary work with recoilless *guns* can be obtained.

In this regard an expression in analogy with the expression for conventional weapons can be written as a ratio for $p_{dn}$ and $p_{pr}$

$$p_{dn} = p_{pr}\left[1 + \frac{1}{2}\frac{\omega}{\varphi_1 q}(1 - \eta)\right],$$

$$p_{av} = p_{pr}\left[1 + \frac{1}{3}\frac{\omega}{\varphi_1 q}(1 - \eta)\right],$$

where    $\eta = \frac{\gamma}{\omega} = \frac{\int_0^t G_{loss}\,dt}{\omega}$.    is the relative gas loss through the nozzle.

Therefore, the state recoilless produces the relation

$$\frac{F_{cr}}{s} = \frac{1}{\varphi \chi \zeta}.$$

Assuming, for instance, $\frac{\omega}{q} = 0.30$, $\zeta = 1.65$, $\chi = 0.85$, $\eta_\Delta = 0.65$,    $1 - \eta_\Delta = 0.35$ we obtain $\varphi = 1.02 + \frac{1}{3} \cdot 0.30 \cdot 0.35 = 1.055$.

Then the ratio of the area of the critical cross section of the nozzle $F_{cr}$ to the cross section of the bore is $s$

$$\frac{F_{cr}}{s} = \frac{1}{1.055 \cdot 0.85 \cdot 1.65} = \frac{1}{1.479} = 0.676 \approx 0.68.$$

Characteristics of the Interior Ballistics of Recoilless Guns

Basic features of firing taking into account characteristics of ballistics of recoilless *guns*. The features of firing from recoilless *guns are* more complex than the features of firing from a conventional *gun* due to the *reasons* described above.

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Approximately 2/3 of the weight of the charge escapes in the form of gas through the nozzle;  together with the *propellant* gases a        fraction of the grain powder is also ejected through the nozzle; these are usually small-grained (with seven *perforations* or short grains with one *perforation*) and are easily carried off with the escaping *propellant* gases.  With a relatively large nozzle diameter the temperature of the *propellant* gases in the chamber and in the bore falls signficantly.  In this case, a fraction of the gases moving the projectile and the fraction of gases escaping with some of the powder through the nozzle, strictly speaking, act under differing conditions; undoubtedly, there is a zone of relative stillness of     gas layers, on the one side of which the gases move in the direction of the projectile, and on the other side of which, the flow is oriented in the direction of the nozzle.

Due to the complexity of the features discussed in solving the problems of interior ballistics, processes discussed

are outlined and simplified by introducing into the solution of *the* basic problem the following assumptions.

1. The process of the flow of *propellant* gases through the nozzle is assumed as stationary and in this case all formula*s* of steady-state motion are used with the introduction of experimental correlation coefficients.

                             there is no
2. It is assumed that                   the ejection of unburned powder grains through the nozzle, although this does take place.
   Therefore, in fact, the consideration of the movement of gas powder mixtures is replaced by the consideration of the movement of only *propellant* gases of the same weight.

3. In case of a large opening for the nozzle, a significant drop in gas temperature is taken into account:

$$\tau = \frac{T}{T_1} < 1.$$

In addition, all previous assumptions for conventional guns remain in force (geometric law of powder burning and its derived relations $\psi = f(z)$ and $\sigma = f_1(z)$, law of the rate of burning $u = u_1 p$, $\theta = \frac{c_p}{c_w} - 1 = const$ and others are assumed).

The system of equations for processes occuring during firing in recoilless guns is presented below.

## The Compilation of an Essential System of equations

Equations for gas formation and the inflow of gases are

$$\frac{d\psi}{dt} = \frac{\varkappa\sigma}{I_\kappa} p,$$

$$\psi = \varkappa z + \varkappa\lambda z^2,$$

$$\sigma = 1 + 2\lambda z = \sqrt{1 + 4\frac{\lambda}{\varkappa}\psi}.$$

Equation for gas loss through the nozzle

$$\eta = \frac{\varphi_2 F_{cr} K_0}{\omega\sqrt{f}} \int_0^t \frac{p\,dt}{\sqrt{\tau}},$$

where $\varphi_2$ is the loss factor for the correlation of calculated and experimental values of $\eta$: $\varphi_2 \approx 0.95.$

In the first period the temperature falls slowly and $\tau = \tau_{av} = const.$ can be assumed. Then

$$\eta = \frac{\varphi_2 F_{cr} K_0}{\omega\sqrt{f_{av}}} \int_{t_0}^t p\,dt,$$

where

$$\int_{t_0}^t p\,dt = I_\kappa (z - z_0).$$

The nozzle is usually closed with a base plate which is calculated in such a way that it is ejected at the beginning of the motion of the projectile at pressure $p_0$; in the closed chamber the pressure $p_0$ corresponds to the values $\psi_0$ and $z_0$. The nozzle is opened and the flow of gases begins at $p = p_0$, $\psi = \psi_0$ and $z = z_0$

Then $\eta = \frac{\varphi_2 F_{cr} K_0 I_\kappa}{\omega\sqrt{f_{av}}} (z - z_0) = \eta_\mu x.$

Equation for the state of gases during the presence of flow. Granting that the free space of the chamber during gas escape (as in mortar) is expressed by the relation

$$W_{\psi_\eta} = W_0 - \frac{\omega}{\delta}(1-\psi) - \alpha\omega(\psi-\eta),$$

the equation for the state of gases is written as follows:

$$p(W_{\psi_\eta} + sl) = RT\omega(\psi-\eta)$$

or

$$ps(l_{\psi_\eta} + l) = f\tau\omega(\psi-\eta),$$

where

$$\tau = \frac{T}{T_1}.$$

Essential Equation for Interior Ballistics. Energy Equation.

First we establish the energy equilibrium during firing.

At a given moment t, the amount of charge $\psi$ was burned, and the amount of energy liberated (energy inflow) is expressed by the relation

$$EQ\omega\psi = \frac{RT_1}{\theta}\omega\psi = \frac{f\omega}{\theta}\psi.$$

This energy is expended:

a) for providing the projectile with the critical energy $\frac{\varphi m v^2}{2}$;

b) to alter the internal energy of the mass of gas in the barrel bore at the given moment which has the temperature $T < T_1$;

$$\frac{RT}{\theta}\omega(\psi-\eta) = \frac{f\omega}{\theta}\tau(\psi-\eta).$$

This value can be replaced by the expression from the equation of state $\frac{ps(l_{\psi_\eta} + l)}{\theta}$;

c) to provide information on the movement of gases escaping through the nozzle at a given moment; in this case one should consider that the energy expended for the transfer and pushing of gases will be greater than the internal energy of the motionless gases if they were to remain in the barrel.

The energy expended for pushing the gases is expressed by

the equation

$$\mathcal{L}c_pT\omega\eta = kEc_{\omega}T\omega\eta,$$

or

$$\frac{f\omega}{\theta}(1+\theta)\tau_{\varkappa\upsilon}\eta_k x = \frac{f\omega}{\theta}D'x,$$

where

$$D' = (1+\theta)\eta_k\tau_{\varkappa\upsilon}.$$

In comparing the inflow and expenditure of energy and transferring $\theta$ in the last term on the rightside, we obtain the equation for energy transformation

$$f\omega\psi = ps(l_{\psi_\eta}+l) + f\omega D'x + \frac{\theta\varphi mv^2}{2}.$$

This can be rewritten in the general form

$$ps(l_{\psi_\eta}+l) = f\omega(\psi - D'x) - \frac{\theta}{2}\varphi mv^2. \qquad (12.11)$$

The value $D'x$ represents the fraction of energy used to eject gases through the nozzle.

5. Equation for the motion of the projectile is

$$ps\,dl = \varphi mv\,dv \qquad (12.12)$$

or

$$ps\,dt = \varphi m\,dv. \qquad (12.12')$$

The system of these equations is solved in a general form using the parameters and functions of Prof. N.F. Drozdov..

Solution of the Essential Problem of Interior Ballistics for Recoilless Guns

Preliminary period (general formulas).
It is assumed that the escape of gases through the nozzle occurs simultaneously with the beginning of the motion of the projectile at the pressure $p_0$:

$$\psi_0 = \frac{\frac{1}{\Delta} - \frac{1}{\delta}}{\frac{f}{p_0} + \alpha - \frac{1}{\delta}}; \quad z_0 = \sqrt{1 + 4\frac{\lambda}{\varkappa}\psi_0}; \quad z_\eta = \frac{\psi_0}{\varkappa}.$$

The first period is $(x = z - z_0)$

$$\psi = \psi_0 + k_1 x + \varkappa\lambda x^2, \qquad (12.13)$$

$$v = \frac{sI_\varkappa}{\varphi m}x. \qquad (12.14)$$

By dividing the equation (12.12) by the expression (12.11), we obtain:

$$\frac{dl}{l_{\psi\eta}+l} = \frac{\varphi m v\, dv}{f\omega(\psi - D'x) - \frac{\theta\varphi m v^2}{2}} = \frac{Bx\,dx}{\psi_0 + (k_1 - D')x - \left(\frac{B\theta}{2} - x\lambda\right)x^2} =$$

$$= -\frac{B}{B_1}\frac{x\,dx}{x^2 - \frac{k_1 - D'}{B_1}x - \frac{\psi_0}{B_1}}.$$

The solution to this differential equation at $Z_{\psi\eta} = Z_{\psi\eta_{av}} = Z_c$, where $\psi_{av} = \frac{\psi_0 + \psi}{2}$ results in the form

$$\ln\left(1 + \frac{l}{l_c}\right) = -\frac{b}{B_1}\ln Z_x$$

or

$$l = l_c(Z_x^{-B/B_1} - 1), \qquad\qquad (12.15)$$

where $Z_x = \int_0^x \frac{x\,dx}{x^2 - \frac{k_1 - D'}{B_1}x - \frac{\psi_0}{B_1}}.$ is the integral of Prof. N.F. Drozdov, in which the escape of gases through the nozzle is taken into account by subtracting D' from the usual variable $k_1$.

As is evident $Z_x = f(\gamma, \beta)$; for the given case taking into account the escape of gases through the nozzle the expression for $\gamma$ and $\beta$ will differ from analogous values given in chapter VII:

$$\gamma = \frac{B_1\psi_0}{(k_1 - D')^2} \quad\text{and}\quad \beta = \frac{B_1}{(k_1 - D')}x,$$

i.e., $\gamma$ and $\beta$ at a given $x$ have other values in comparison with the variables $\gamma$ and $\beta$ for conventional guns under the same charge conditions since the measure of gas loss D' is subtracted from $k_1$.

Gas pressure is calculated from the basic equation

$$p = \frac{f\omega}{s}\frac{\psi - D'x - \frac{B\theta}{2}x^2}{l_{\psi\eta}+l}. \qquad\qquad (12.16)$$

In this case also the value D'x is subtracted from $\psi$ which makes this formula different from the basic one.

The value $x_m$ is found using the formula, which is analogous to the basic one: $\quad x_m = \dfrac{k_1 - D'}{2B_1 + B}$.

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad ... \prime$

By using $x_m$ and the formulas (12.13)-(12.16), $\psi_m$, $v_m$, $l_m$ and $P_{max}$ are calculated. Using the value of $x_x = 1-2.$ and the same formulas the elements for the first end of the period can be found ($\psi_k = 1$): $v_f$, $l_k$, $\rho_k$. These values are also the initial ones for the second period.

The second period is the adiabatic expansion of $\overset{\text{propellant}}{\phantom{xx}}$ gases concurrent with the escape of gases through the nozzle. This period is characterized by the fact that all the powder has burned($\psi = 1$) but the escape of gases continues simultaneously with the movement of a fraction of gases after the projectile: the relative fraction of gases escaping at a given moment $\eta = y/\omega$ continues to increase and $\eta > \eta_k$; the velocity of the projectile also continues to increase: $v > v_f$.

For the solution we have a system of two equations:
$$ps\, dt = \varphi m v\, av, \qquad\qquad (12.12)$$
$$ps(l_{1\eta} + l) = f\omega(1 - D'x) - \frac{\theta\varphi m v^2}{2}. \qquad\qquad (12.11)$$

Since the loss of gases through the nozzle
$$D'x = (1+\theta)\,\eta_k x = (1+\theta)\,\frac{\varphi_2 l_{\epsilon} \rho K_0}{\omega\sqrt{f\omega}}\,l_k x$$
is proportional to the $\overset{\text{momentum}}{\phantom{xx}}$ of the pressure $I - I_0$, counted from the beginning of the motion of the projectile, and the velocity of the projectile v is also proportional to the value $I - I_0 = \int_{\psi_0}^{\psi}$ regardless of whether the powder burned, then the value of v is proportional to x in the second period also. Therefore, the system of equations (12.12) and (12.11) for the second period can be solved using the method for the first period; only a slightly different expression for the integral of the trajectory is obtained. By dividing the equation (12.12) by (12.11), we obtain:
$$\frac{dl}{l_{1\eta} + l} = \frac{\varphi m v\, dv}{f\omega\left[1 - D'x - \dfrac{v^2}{v^2}\right]} = B\,\frac{x\, dx}{1 - D'x - \dfrac{B\theta}{2}x^2},$$

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since

$$\frac{v^2}{v_{m\,p\,r}^2}=\frac{B\theta}{2}x^2.$$

In this case $B_1=\frac{B\theta}{2}$ and $\frac{B}{B_1}=\frac{2}{\theta}$,

$$\frac{dl}{l_{1\eta}+l}=-\frac{2}{\theta}\cdot\frac{x\,dx}{x^2+\frac{D'}{B_1}x-\frac{1}{B_1}}.$$

Designating $\frac{B_1}{D'}x=\beta$ and $\frac{B_1}{D'^2}=\gamma$ and multiplying the numerator and denominator on the right hand side by $\frac{B_1^2}{D'^2}$, we obtain

$$\frac{dl}{l_{1\eta}+l}=-\frac{2}{\theta}\cdot\frac{\beta\,d\beta}{\beta^2+\beta-\gamma}=-\frac{2}{\theta}d\ln Z.$$

By integrating the preceding equation, we obtain

$$\frac{l_{1\eta_{a\dot{v}}}+l}{l_{1\eta_{a\dot{v}}}+l_k}=\left(\frac{Z}{Z_K}\right)^{-\frac{2}{\theta}},$$

where

$$Z=\int_0^\beta\frac{\beta\,d\beta}{\beta^2+\beta-\gamma};\quad Z_b=\int_0^{\beta_k}\frac{\beta\,d\beta}{\beta^2+\beta-\gamma}.$$

Finally we have an association between $l$ and $x$(or $v$) in the form of

$$l=(l_{1\eta_{a\dot{v}}}+l_k)\left(\frac{Z}{Z_k}\right)^{-\frac{2}{\theta}}-l_{1\eta_{a\dot{v}}}\qquad(12.17)$$

The formula obtained is of the same type as the formulas for the first period but the function Z differs from the integral of Prof. N.F. Drozdov $Z_x$ in that the input values $\gamma$ and $\beta$ differ from the integral $Z_x$ and in addition, in the integrand expression in the numerator the value $\beta$ has a "+" and not a "-" sign . This integral can be calculated by the same method as the integral $Z_x$.

After analogous transformations, we obtain

$$Z=\left(1-\frac{\beta}{b-1}\right)^{\frac{b-1}{b}}\left(1+\frac{\beta}{b+1}\right)^{\frac{b+1}{b}},$$

where

$$b=\sqrt{1+4\gamma}\quad\text{and}\quad\gamma=\frac{B_1}{D'^2}=\frac{B\theta}{2D'^2}.$$

Pressure in the second period is calculated by the formula

$$p = \frac{f\omega}{s} \cdot \frac{1 - D'x - \frac{B\theta}{2}x^2}{l_{1\eta} + l}, \qquad (12.18)$$

where

$$x = x_\kappa \frac{v}{v_\varphi}.$$

The typical curve for gas pressure p, l in the bore of a recoilless *gun* is peaked; the powder burns rapidly at low value and in the se*cond* period the curve of adiabatic expansion with loss falls rapidly; muzzle pressure is insignificant (Fig. 12.7).
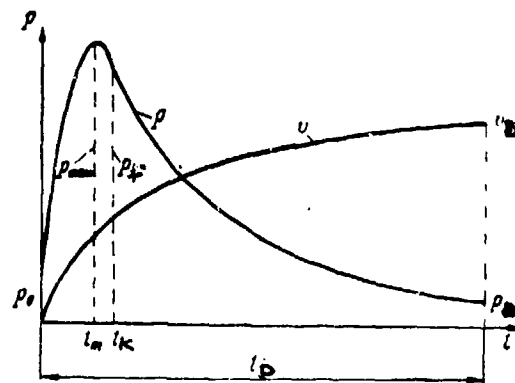


**Fig. 12.7.** Curves p, l and v, l for recoilless weapon

12.3 Features of Interior Ballistics for *Mortar*

Features of Firing from a *Mortar*

In comparison with firing from a conventional artillery weapons, firing from *mortar* has several features.

1. The first characteristic is the design and the placement of the charge. A diagram for the design of *mortar* is presented below (Fig. 12.8).

The *primary* charge of the *mortar* is placed in a cardboard cartridge (barrel sleeve), placed in the tube of the stabilizer 1 (tail end of the *projectile*). The tube has 4 or 6 |vents|circular|2, through which the *propellant* gases f( ned within the barrel sleeve after piercing of the cardboard should cape into the space behind the mine and ignite the *booster* charge ω.

During ignition the *projectile* is lowered in the bore expelling the air through the gap 3. The capsule of the cartridge with the primary charge hits the fuse 4 fastened to the bottom of the

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*mortar*

bore; the capsule and the powder charge are ignited; at this time the powder burns initialy in the closed space of the cartridge at fairly high density of ignition $\Delta_0 = 0.50 - 0.60$. At a certain moment the gas pressure *pierces* the wall of the cardboard sleeve and the gases escape through the *vents* 2 in the tube of the stabilizer into the space of the chamber $W_0$ (*Projectile* space).

Under such conditions of very rapid burning of the very fine, small powder the greatest pressure of gases within the tube of the stabilizer, as experiments showed, depends to a significant extent on the size of the vents in the tube, the thickness of the barrel sleeve wall, and on the temperature of the charge. As a result of slight difference in pressure at which the sleeve walls are penetrated, the values of the greatest pressure within the tube of the stabilizer can be widely scattered.
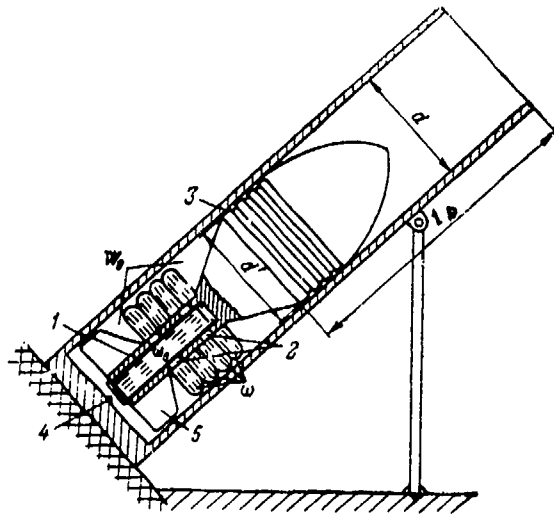


Fig. 12.8. Diagram of the design of *mortar*

Consequently, the value of the composition, weight of the capsule igniter, and the rapidity of powder burning are more significant in the *mortar* than in *guns* the greater the igniting *momentum* is, the more uniform is the ignition of the powder.

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2. The following feature is the fact that the gases of the primary charge $\omega_0$, burning initially within the chamber of the stabilizer at $\Delta =0.50-0.60$, escape into the *projectile space* ; expand rapidly, and cool. Since the surface of the fin stabilizers 5 and the bottom part of the mortar shell is large and the density of the ~~ammunition~~ ~~charge of~~ the primary charge with respect to the entire volume of the chamber $W_0$ ($\Delta \approx 0.01$) is small; there is a great loss for heat transfer to the walls of the bore and ... - *projectile* . This loss is even greater as a result of the slow movement of the ~~shell~~ *projectile* and the long intermediate time during which the gases are in contact with the walls of the mortar.

If there are *booster* charges $\omega$ , then the powder within them *is* ignited by the gases of the primary charge and the movement of the *projectile* proceeds under the effect of the total pressure of gases of the primary and booster charges.

3. The third feature of firing from mortar is the bursting through of the gases through the gap between the *projectile* . and the bore. Because of this gap 3 between the *projectile* and the bore walls, a fraction of the gases will burst through this gap at the beginning of *projectile* movement and subsequently their energy is not used.

In *valve* mortars with open remote *valve* a significant amount of gases also escaped through the *valve* . The loss of gases through the gap and the *valve* are included in the foundations of general relations in gas dynamics.

As high-speed photography shows, a signficant fraction of the gases escape from the barrel bore of the mortar before the *projectile* appears from the bore, accompanied by the release of most of the gases. This fraction of the gases which bursts through the gap and *is* not responsible for creating speed for the shell, composes from 10-15% of the entire quantity, while in conventional *guns* the fraction of gases escaping from the gap between the driving band and the perforations is insignificant.

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4. The fourth feature is that the pressure of the *ejection* is almost equal to zero. Just so in a smooth bore, there is no loss of energy for overcoming the friction and the rotation of the shell.

Thus, the solution of the problem of interior ballistics, on the one hand, is simplified by the fact that the pressure of the *ejection* and some of the secondary work is taken as equal to zero, and on the other, it is complicated as a consequence of the necessity to include the large loss for heat transfer and loss of gases through the gap.

While in firing a conventional mortar with support in an end plate recoil is essentially nonexistant and the relative weight of the charge $\frac{\omega}{q}$ is very small (0.01-0.02 during full charge), then in practice the coefficient $\varphi = 1$ can be used.

To maintain unity of procedure and designation of parameters and functions further presentations on the *solution* of basic problems of interior ballistics for mortar are given *using* the designations of Prof. P.F. Drozdov for conventional artillery weapons.

Analytical Solutions of Basic Problems of Smoothbore Mortars
 (Simplified Method of Prof. M.E. Serebryakov)

The following assumptions are made as the basis for the analytical solution.

1. The pressure of the *ejection* is absent. A mortar has a circular gap between the shell and the bore; its area is $s_{gap}$.

2. The burning of the primary charge within the tube of the stabilizer is not considered.

Gases of the primary charge escaping from the tube of the stabilizer into the *projectile* space, creates the pressure $p_0$, at which the powder of the booster charge *ignites*. In this way the

the primary charge is the igniter of the booster charge.

3. The ignition of the booster charge is instantaneous and simultaneous for all grains and at all points on the surface of each grain.

4. The burning of the grains of the booster charge proceeds in parallel layers according the geometric law of burning and is expressed by existing formulas:

$$\psi = \varkappa z + \varkappa \lambda z^2;$$
$$\sigma = 1 + 2\lambda z.$$

5. The rate of the burning of the powder is proportional to the pressure (to the power of one): 

$$u = \frac{de}{dt} = u_1 p,$$

where $u_1$ is the rate of burning at $p=1$.

6. The motion of the *projectile* begins at pressure $p_0$ concurrently with the initiation of burning of the booster charges (at $p=p_0$, $\psi=0$; $1=0$; $v=0$).

7. The escape of gases through the gap begins with the initiation of burning of the booster charges and with the movement of the *projectile*.

8. The total *momentum* of the rise in pressure $\int_0^{t_b} p\, dt = \frac{e_1}{u_1}$ does not depend on the density of the charge $\Delta$ and on the value of the initial pressure $p_0$ at which the powder ignites.

9. The loss of gases through the gap according to general formulas of gas dynamics is proportional to the *momentum* of the increase in pressure:

$$Y = \omega \eta = \zeta' A s_{gap} \int_0^t p\, dt = \zeta' A s_{gap} I,$$

where $\eta$ is the fraction of gases escaping through the gap;

A is the loss factor:

$$A = \left(\frac{2}{k+1}\right)^{\frac{1}{k-1}} \sqrt{\frac{2gk}{k+1}\frac{1}{f}};$$

$\zeta'$ =loss coefficient characterizing the form of the vent or gap; for a round vent $\zeta$ =0.95; for a crescent shaped slot $\zeta \approx 0.66$;

$S_{gap}$ =area of the cross section of the gap.

10. Heat transfer was considered in experiments using a special _installation_ in which the primary charge burns under the same conditions as in mortar.

By determining the greatest gas pressure of the primary charge $p_{max0}$ using this experimental _installation_ , we find the force of the powder $f_0$ of the primary charge, taking into account the cooling of the gases due to their escape from the tube of the stabilizer into the shell volume and due to the heating of the bore walls and the fins of the stabilizer:

$$f_0 = p_{max0} \left( \frac{1}{\Delta_0} - \alpha_0 \right),$$

where $\Delta_0$ =density of the charge of the primary charge with respect to the entire _projectile space_ $W_0$ (chamber).

The value of $f_0$ is much less than the force $f$ of the booster charges from powder of the same nature, which is determined by experiments in a conventional manometric bomb.

Additional heat transfer during the movement of the shell is negligible (since the bore walls and shell are heated by gases escaping through the circular gap and outdistancing the shell) or is calculated indirectly. Since the gases do not _perform_ work to rotate the shell, to overcome friction against the grooves, and for the recoil, and $\frac{q}{q}$ =1%, then the coefficient for calculating secondary work may be assumed equal to one.

During firing from the mortar the following periods may be delineated:

1) burning of the primary charge up to the piercing of the

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opening into the sleeve and the escape of gases into the chamber;
this phase with the given assumptions is analogous to the preli-
minary period;

2) the first period corresponds to the burning of the booster
charges concurrent with the escape of a fraction of the gases
through the gap (l varies from 0 to $l_b$);

3) the second period is the expansion of gases formed in the
first period concurrent with the escape their through the gap
$s_{gap}$.

The movement of the projectile and the escape of gases through
the gap (on the basis of the given assumptions) begins with the
pressure $p_0$, created by the gases of the primary charge; $p_0$
can be calculated in the experimental installation:

$$p_0 = \frac{f_0\omega_0}{W_0 - \frac{\omega}{\delta} - \alpha\omega_0} = \frac{f\Delta_0}{1 - \frac{\Delta}{\delta} - a_0\Delta_0},$$

where $\Delta = \frac{\omega}{W_0}$

If $s$ designates the cross section of the mortar bore:

$$s = \frac{\pi}{4} d^2;$$

and $s'$ designates the area of the cross section of the projectile

at its central largest diameter; $s' < s$, then the area of the
gap between the projectile and the bore walls is $s_{gap} = s-s'$.

The velocity of the projectile is determined by the motion equa-
tion $\varphi m \, dv = s'p \, dt,$

$$v = \frac{s'}{\varphi m} \int_0^t p \, dt = \frac{s'}{\varphi m} I = \frac{s' I_\kappa}{\varphi m} z,$$

where

$$z = \frac{I}{I_\kappa} = \frac{e}{e_1}.$$

To solve the problem The following system of
equations can be used.

1. The basic equation for pyrodynamics taking into account

the escape of a fraction of the gases through the gap and loss
for heat transfer

$$sp(l'_\psi + l) = f_0 \omega_0 + f \omega \psi - f'Y - \frac{\theta}{2} \varphi m v^2,$$

where

$$l'_\psi = \frac{1}{s} \left[ W_0 - \frac{\omega}{\delta} (1 - \psi) - \alpha (\omega \psi - Y) - \alpha_0 \omega_0 \right]$$

includes the loss of gases through the gap s-s'.

The value $f'$ is the force of the gas mixture of the primary
and booster charges; essentially, it varies with burning time of
the booster charges from $f_0$ to $\frac{f_0 \omega_0 + f \omega}{\omega_0 + \omega} < f,$ with the interme-

diate value $f' = \frac{f_0 \omega_0 + f \omega \psi}{\omega_0 + \omega \psi}.$

Since the heat transfer to the walls during the first period
is not yet directly accounted for, it can be indirectly included
by taking the value f of the booster charges as greater than f'.
In this case the basic equation is rewritten as follows:

$$sp(l'_\psi + l) = f_0 \omega_0 + f (\omega \psi - Y) - \frac{\theta}{2} \varphi m v^2. \qquad (12.19)$$

2. The equation for the movement of the $\overline{\text{lis}\,) \text{projectile}}$

$$s' p d l = \varphi m v d v \qquad (12.20)$$

3. The law for the burning of powder (geometric) for small
flake powder is $\psi = \chi z + z \lambda z^2.$

4. The formula for the velocity of the $^{\text{projectile}}$ is

$$v = \frac{s' I_b}{\varphi m} z.$$

5. The relative loss of gases is

$$\eta = \frac{Y}{\omega} = \frac{\zeta' A s_{gap} l_\kappa}{\omega} z = \eta_\kappa z, \qquad (12.21)$$

where

$$\eta_\kappa = \frac{\zeta' A s_{gap} l_\kappa}{\omega} = \frac{\zeta' A s_{gap}}{\omega} \frac{e_1}{u_1};$$

$\zeta' < 1$ =coefficient for the shape of the vents;

$\eta_\kappa$ =relative loss of gases at the end of the burning of the
powder.

Let us introduce the designation:

$$B' = \frac{s' I_{\mathbf{k}}^2}{f \omega \varphi m} = \left(\frac{s'}{s}\right)^2 \frac{s^2 I_{\mathbf{k}}^2}{f \omega \varphi m} = \left(\frac{s'}{s}\right)^2 B;$$

$$\chi_0 = \frac{f_0 \omega_0}{f \omega} = \text{relative energy of the primary charge.}$$

By substituting in equation (12.19) the variables $\psi, \nu$ and $Y$ (or $\eta$) by their expression through $z$, we obtain a basic equation of interior ballistics in the following form:

$$sp(l'_\psi + l) = f\omega \left[ \chi_0 + \varkappa z + \varkappa \lambda z^2 - \eta_\mathbf{k} z - \frac{B'\theta}{2} z^2 \right] =$$

$$= f\omega \left[ \chi_0 + (\varkappa - \eta_\mathbf{k}) z - \left( \frac{B'\theta}{2} - \varkappa \lambda \right) z^2 \right]. \qquad (12.22)$$

From equation (12.20) we have

$$s' p \, dl = \frac{s'^2 I_{\mathbf{k}}^2}{\varphi m} z \, dz. \qquad (12.20')$$

By dividing (12.20') term by term by (12.22), we obtain

$$\frac{s'}{s} \frac{dl}{l'_\psi + l} = B' \frac{z \, dz}{\chi_0 + k'_1 z - B'_1 z^2} = -\frac{B'}{B'_1} \frac{z \, dz}{z^2 - \frac{k'_1}{B'_1} z - \frac{\chi_0}{B'_1}} = -\frac{B'}{B'_1} d \ln Z,$$

where $\quad k'_1 = \varkappa - \eta_\mathbf{k}; \quad B'_1 = \frac{B'\theta}{2} - \varkappa \lambda; \quad A_s = \frac{s}{s'} \frac{B'}{B'_1} .$

$Z$ is the known function of Prof. N.F. Drozdov. Thus, we obtain

$$\frac{dl}{l'_\psi + l} = -A_s \frac{z \, dz}{\xi_1(z)} = -A_s d \ln Z. \qquad (12.23)$$

The equation (12.23) can be solved exactly by the method of Prof. N.F. Drozdov by reducing it to a linear differential equation of the first order, but taking into account that the density of the charge in the $^{projectile}$ is small ($\Delta < 0.15$) and that consequently, $l'_\psi$ varies little, one can assume an average value for $\ell'_\psi = \ell'_{\psi_{av}}$ .After integration a simple expression for the shell projectile is obtained in the form

$$\frac{l'_{\psi_{av}} + l}{l'_{\psi_{av}}} = 1 + \frac{l}{l'_{\psi_{av}}} = Z^{-A_s},$$

from which follows

$$l = l'_{\psi_{av}}(Z^{-A_s} - 1). \qquad (12.24)$$

lg $Z^{-1}$ is calculated according the to the table of N.F. Drozdov (cf. page 417) with the initial values $\quad \gamma = \frac{B'_1 \chi_0}{k_1'^2}; \quad \beta = \frac{B'_1}{k'_1} z.$

Consequently, the solution to the problem for the first period for the mortar differs from the solution for artillery weapons only in that at z there is the coefficient $k_1 = \varkappa - \eta_\varkappa$ in place of $l_1 = \varkappa \sigma_0$, the value $\chi_0 = \frac{f_0 \omega_0}{f \omega}$ in place of $\psi_0$, and the value $B' = \theta \left(\frac{s'}{s}\right)^2$ in place of B. These features of firing from mortar express themselves in the values $\gamma = \frac{B_1' \chi_0}{k_1'^2}$ and $\beta = \frac{B_1'}{k_1'} z$, which in turn increases the value $z^{-1}$ and the path $l$ and decreases the value of the pressure p in comparison with the pressure during the absence of gas escape through the gap.

The pressure p is calculated using the formula obtained from equation (12.19):

$$p = \frac{f\omega}{s} \frac{\chi_0 + \psi - \eta - \frac{\theta}{2} \frac{\varphi m v^2}{f\omega}}{l_\psi' + l} = \frac{f\omega}{s} \frac{\chi_0 + k_1' z - B_1' z^2}{l_\psi' + l} . \quad (12.25)$$

To determine $z_m$, corresponding to the greatest gas pressure $p_{max}$, we differentiate this expression with respect to z and using the equation (12.23), after a series of transformations we obtain

$$z_m = \frac{\varkappa\left(1 + \frac{p_{max}}{f\delta_1}\right) - \eta_k\left(1 + \frac{\alpha p_{max}}{f}\right)}{B'\left(\frac{s}{s'} + \theta\right) - 2\varkappa\lambda\left(1 + \frac{p_{max}}{f\delta_1}\right)} = \frac{\varkappa - \eta_k\left(\frac{1 + \frac{\alpha p_{max}}{f}}{1 + \frac{p_{max}}{f\delta_1}}\right)}{\frac{B'\left(\frac{s}{s'} + \theta\right)}{1 + \frac{p_{max}}{f\delta_1}} - 2\varkappa\lambda},$$

where $\frac{1}{\delta_1} = \alpha - \frac{1}{\delta}$ .

At $\eta_k = 0$ and s=s', this formula becomes the general formula for $z_m$.

If $z_m \leqslant 1$, we have a real maximum pressure; if $z_m > 1$, the maximum is unreal, and in this case, the greatest actual pressure will occur at the end of burning $p_f$:

$$p_f = \frac{f\omega}{s} \frac{1 + \chi_0 - \eta_k - \frac{B'\theta}{2}}{l_1' + l_k},$$

where $\quad l_1' = l_0[1 - a\Delta(1 - \eta_{\kappa})] \quad \Delta_{\kappa} = \dfrac{\omega}{W_0}(1 - \eta_{\kappa}) = \Delta(1 - \eta_{\kappa}).$

The remaining factors at the end of the first period will be $\quad v_{\phi} = \dfrac{s' I_{\kappa}}{\varphi m}; \quad l_{\kappa} = l_{\psi}'(Z_{\kappa}^{-A_s} - 1),$

where $\quad \beta_{\kappa} = \dfrac{B_1'}{k_1'}.$

For solving the problem of the second period, when $\psi = 1$, there is the following system of equations:

$$sp(l_1' + l) = f_0 \omega_0 + f\omega(1 - \eta_{\kappa} z') - \dfrac{\theta}{2}\varphi m v^2, \qquad (12.26)$$

$$s'p\,dl = \varphi m v\,dv, \qquad\qquad (12.20)$$

where

$$z' = \dfrac{I}{I_{\kappa}} = \dfrac{\int_0^t p\,dt}{\int_0^{t_{\kappa}} p\,dt} = \dfrac{v}{v_{\phi}},$$

in this case $z'$ is greater than one.

The value $\eta_{\kappa} z'$ is also accounted for in the second period of gas emission from the gap. As in the first period, the total loss is proportional to the momentum of the gas pressure, which in turn, is proportional to the velocity of the projectile.

Equation (12.26) can be rewritten as:

$$sp(l_1' + l) = f\omega\left[\chi_0 + 1 - \dfrac{\eta_{\kappa}}{v_{\kappa}}v - \dfrac{v^2}{v_{pr}^2}\right], \qquad (12.26')$$

where

$$\eta_{\kappa} = \dfrac{\zeta' A s_{pr} I_{\kappa}}{\omega}; \quad v_{\phi} = \dfrac{s' I_{\kappa}}{\varphi m},$$

$$v_{pr}^2 = \dfrac{2f\omega}{\varphi\theta m};$$

$$\dfrac{\eta_{\kappa}}{v_{\phi}} = \dfrac{\zeta' A s_{pr} I_{\kappa}}{\omega}\dfrac{\varphi m}{s' I_{\kappa}} = \zeta' A \dfrac{\varphi}{g}\dfrac{\eta}{\omega}\dfrac{s_{pr}}{s'} = \eta_{\kappa}'.$$

Dividing (12.20) by (12.26'), we obtain:

$$\dfrac{dl}{l_1' + l} = \dfrac{s}{s'}\dfrac{\varphi m}{f\omega}\dfrac{v\,dv}{1 + \chi_0 - \eta_{\kappa}' v - \dfrac{v^2}{v_{pr}^2}} = -\dfrac{s}{s'}\dfrac{2}{\theta}\dfrac{v\,dv}{v^2 + \eta_{\kappa}' v_{pr}^2 v - (1 + \chi_0)v_{pr}^2}$$

or

$$\dfrac{dl}{l_1' + l} = -\dfrac{s}{s'}\dfrac{2}{\theta}\dfrac{v\,dv}{v^2 + \eta_2 v - \eta_3}, \qquad (12.27)$$

where $\quad \eta_2 = \eta_{\kappa}' v_{pr}^2 = 2\zeta' A \dfrac{f}{\theta}\dfrac{s_{pr}}{s'} = \dfrac{\eta_{\kappa}}{\dfrac{v_{\phi}}{v_{pr}^2}} = \dfrac{\eta_{\kappa}}{v_{\phi}}v_{pr}^2,$

$$\eta_3 = (1+\chi_0)\, v_{\text{пр}}^2 = (1+\chi_0)\,\frac{2f\omega}{\varphi\theta m} = v_{\text{пр}}'^2 .$$

By integrating (12.27) we obtain

$$\int_{l_\kappa}^{l} \frac{dl}{l_1' + l} = -\frac{s}{s'}\,\frac{2}{\theta}\int_{v_\phi}^{v}\frac{v\,dv}{v^2 + \eta_2 v - \eta_3}, \qquad (12.28)$$

$$\int_{l_\kappa}^{l}\frac{dl}{l_1' + l} = \ln\frac{l_1' + l}{l_1' + l_\kappa}. \qquad (12.29)$$

The integral of the right side is found by expanding the integrand function to a partial fraction by the method proposed by Prof. N.F. Drozdov.

Solve for the roots of the equation $v^2 + \eta_2 v - \eta_3 = \xi'(v) = 0$:

$$v = -\frac{\eta_2}{2}\left(1 \pm \sqrt{1 + 4\frac{\eta}{\eta_2^2}}\right) = -\frac{\eta_2}{2}(1 \pm b),$$

where

$$\frac{\eta_2}{2} = \frac{f}{\theta}\,\frac{s_{\text{сн}}}{s'}\,\zeta' A; \quad b = \sqrt{1 + 4\frac{\eta_3}{\eta_2^2}} = \sqrt{1+4\gamma};$$

$$\gamma = \frac{\eta_3}{\eta_2^2} = \frac{(1+\chi_0)v_{\text{пр}}^2}{\eta'^2(v_{\text{пр}}^2)^2} = \frac{1+\chi_0}{v_{\text{пр}}^2\eta_\kappa'^2}, \quad \text{где } \eta_\kappa' = \frac{\eta_\kappa}{v_\phi};$$

$$\gamma = \frac{(1+\chi_0)}{\eta_\kappa}\left(\frac{v_\phi^2}{v_{\text{пр}}^2}\right) = \frac{(1+\chi_0)}{\eta_\kappa}\,\frac{B'\theta}{2};$$

$$v_1 = -\frac{\eta_2}{2}(1+b); \quad v_2 = -\frac{\eta_2}{2}(1-b) = \frac{\eta_2}{2}(b-1);$$

$$\eta_\kappa = \frac{\zeta' A s_{\text{сн}} l_\kappa}{\omega};$$

$$v_2 - v_1 = \eta_2 b; \quad \frac{v}{\xi'(v)} = \frac{A_1}{v - v_1} + \frac{A_2}{v - v_2};$$

$$A_1 = \frac{b+1}{2b}; \quad A_2 = \frac{b-1}{2b};$$

$$\int_{v_\phi}^{v}\frac{v\,dv}{\xi'(v)} = \frac{b+1}{2b}\int_{v_\phi}^{v}\frac{dv}{v - v_1} + \frac{b-1}{2b}\int_{v_\phi}^{v}\frac{dv}{v - v_2} =$$

$$= \ln\left(\frac{v - v_1}{v_\kappa - v_1}\right)^{\frac{b+1}{2b}}\left(\frac{v - v_2}{v_\kappa - v_2}\right)^{\frac{b-1}{2b}} = \ln\frac{Z_v'}{Z_{v_\phi}'}. \qquad (12.30)$$

By substituting the expressions (12.29) and (12.30) into (12.28), we obtain

$$\left(\frac{l_1'+l}{l_1'+l_\kappa}\right)^{-\frac{s'}{s}\frac{\theta}{2}}=\left(\frac{l_1'+l_\kappa}{l_1'+l}\right)^{\frac{s'}{s}\frac{\theta}{2}}=\left(\frac{v-v_1}{v_\ast-v_1}\right)^{\frac{b+1}{2b}}\left(\frac{v-v_2}{v_\ast-v_2}\right)^{\frac{b-1}{2b}}=\frac{Z_v'}{Z_{v_\ast}'}$$

or we finally have

$$\left(\frac{l_1'+l_\kappa}{l_1'+l}\right)^{\frac{s'}{s}\theta}=\left(\frac{v-v_1}{v_\ast-v_1}\right)^{\frac{b+1}{b}}\left(\frac{v-v_2}{v_\ast-v_2}\right)^{\frac{b-1}{b}},$$

$$l=l_1'\left[\left(1+\frac{l_\kappa}{l_1'}\right)\left(\frac{Z_v'}{Z_{v_\ast}'}\right)^{-\frac{s}{s'}\frac{2}{\theta}}-1\right]. \qquad (12.31)$$

Using this equation, given the values $v > v_\ast$, we first find the value of the left hand side and then the corresponding value of the trajectory of the projectile $l$; by constructing a graph, we find the value $v_b$ corresponding to the value $l_b$ by interpolating or graphically, and as a check control we once again at the value $v = v_b$.

Pressure is calculated by the formula

$$p=\frac{f\omega}{s}\frac{\left(1+\chi_0-\frac{\eta_\kappa}{v_\ast}v-\frac{v^2}{v_\ast^2}\right)}{l_1'+l}. \qquad (12.32)$$

Example. The calculation of ballistice elements for an 82 mm mortar

### Input Data

$W_0 = 0,720;$  $l_b = 10,20,$  $s = 0,5277;$  $s_{gap} = 0,0082;$  $f = 1120 \cdot 10^3;$

$q = 3,4;$  $l_0 = 1,363;$  $\omega_0 = 0,0072;$  $\omega = 0,0366;$  $\alpha = 0,85;$

$f_0 = 679 \cdot 10^3;$  $\Lambda_b = 7,48;$  $\varkappa\lambda = -0,255;$  $l_\kappa = 55;$  $A\xi_1' = 0,004.$

$\theta = 0,15;$  $c_q = 6,16;$  $A = 0,006;$  $\xi_1' = 0,666;$

$\varkappa = 1,255;$  $\Delta = 0,0608;$  $\varphi = 1;$

$\delta = 1,64;$

$\xi_1'$ is the coefficient for the shape of the vents; its value 0.665 is from data from experiments in a special device.

### Computation of Constants

$\chi_0 = 0,1192;$  $\eta_\kappa = 0,04923;$  $B' = 0,5923;$

$B_1' = 0,2994;$  $\frac{B'}{B_1'} = 1,979;$  $\gamma = 0,02452;$  $\beta_\kappa = 0,2483.$

Elements of firing: $l_m = l_K = 0.700$ dm; $P_b = 48$ kg/cm$^2$; $P_f = P_{max} = 398$ kg/cm$^2$; $v_b = 205.5$ m/sec.

These results are similar to experimental data $p_{max} = 380$ to 390, $v_b = 202$ to 205.

With the same constants and $\theta = 0.20$, $P_f = p_{max} = 392$; $= 201.0$.

At $\theta = 0.18$ a better correlation between calculated and experimental results would have been obtained.



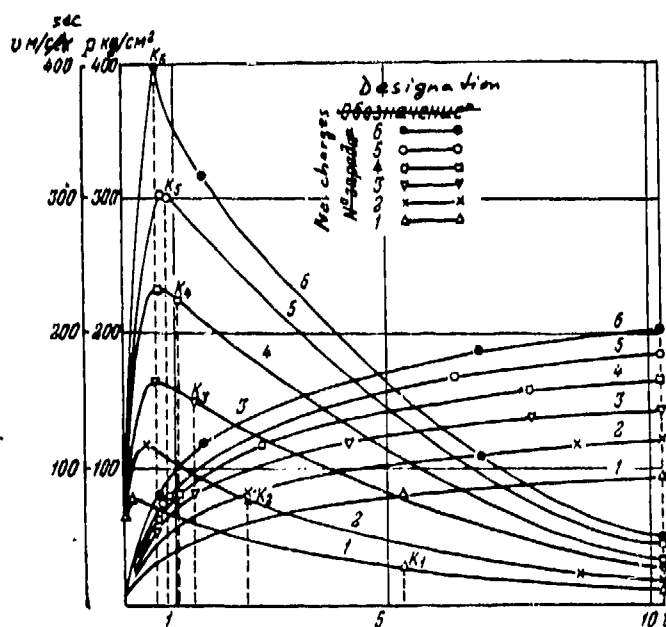Fig. 12.9. Curves p, l and v, l calculated for a mortar with different charges

The results presented indicate that the analytical formulas derived above enable with good accuracy the calculation of ballistic elements of firing from mortar ($P_f$, $P_{max}$, $l_f$, $l_m$, $v_f$, $v_m$, $P_b$, $v_b$) and the construction of curves for the pressure of propellant gases and the velocity of the shell as a function of its trajectory.

If there is only a primary charge $\omega_o$ and no booster charges $\omega$, the problem is solved as in cases of instant burning of the charge taking into account heat transfer due to the decrease in the force of the powder f to $f_O$, determined by using a special installation or by theoretical computations.

Fig. 12.9 presents curves p, 1 and v, 1 calculated for a mortar with different charges. With the decrease in the charge, the cessation of burning of the powder is shifted to the muzzle face and $p_{max}$ and $v_O$ decrease.

Concepts on Ballistic Design of Mortars

The purpose of the ballistic design of mortar is the calculation of the dimensions of the barrel bore and the conditions of ignition necessary to fire a projectile of a given caliber and weight at a specific speed in the muzzle face. It is necessary to find the volume of the chamber $W_O$, the length of the trajectory in the bore $\ell_\delta$, weight of the charge $\omega$, the thickness of powder $2e_1$, and the value of the maximum pressure $p_{max}$. In addition, additional conditions can be created, for instance the value of the pressure $p_{max}$ and others.

The problem of mortar desing, as is the case for grooved bores, is indefinite and allows many solutions.

Prof. G.V. Oppokov recommends the following procedure for the ballistic design of mortars.

Initially, the values of $p_{max}$ and $\Delta$ are calculated based on the values of the power factor
$$C_* = \frac{q v_\delta^2}{2 g d^3} = c_q \frac{v_\delta^2}{2g}$$
$$P_{max} = 30 C_*; \quad \Delta = 0{,}0045 C_* \text{ or } \Delta = 0{,}00015 p_{max} \; kg/cm^2,$$
where $C_*$ is given in m.m/dm$^3$.

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The ballistic calculations for the bore are derived for the greatest charge. The value $\eta_0$ for full charges is small, since the pressure curve for a full charge is peaked and the burning ends near $p_{max}$; $\psi \approx 1$. For a 120 mm mortar $\eta_0 = 132$ t·m/kg, for a 107 mm 130 t·m/kg, and for a 82 mm 166 t·m/kg.

In most cases in mortar design the volume of the chamber $W_0$ has an already previously assigned volume, since it is determined by the dimensions and shape of the tail end of the *projectile* during its lowering to the bottom, when the capsule touches the *firing pin*

After collection of data on existing shapes and their analysis further calculations are made in the following sequence:

1) $E_{b} = \dfrac{mv_b^2}{2}$ ;

2) $C_{i} = \dfrac{E_b}{d^3} = q\dfrac{v_b^2}{2g}$ ;

3) $p_{max} = 30 C_i$ кг/см²;

4) $\Delta = 0{,}0045 C_i$ кг/дм³;

5) $W_0$ *is known.*

Then

6) $\omega = \Delta W_0$ ;

7) $\eta_\omega = \dfrac{2v_b^2}{2g\omega} = \dfrac{E_b}{\omega}$ ;

8) $v_{pr} = \sqrt{\dfrac{2gf}{\varphi\theta}\dfrac{\omega}{q}}$ ;

9) $p_{pr} = \dfrac{f\Delta}{1 - \dfrac{\Delta}{\delta}}$ * .

Subsequently, being given the value $p_{max}/p_{pr}$ one uses special tables[**] and finds the values $B$, $\Lambda_\delta = \dfrac{\ell_\Delta}{\ell_0}$ , $\Lambda_v = \dfrac{v_{m\,тab}}{v_f\,T_{ab}}$ , $\eta = \dfrac{p_{av}}{p_{max}}$ and other ballistic elements. Having obtained in the results of the calculations of the first variant at several values of $p_{max}$ and $\Delta$, specific values $\ell_\Delta$ and other characteristics, one must in additional calculations use 2-3 more values of $\Delta$, near $\Delta$ of the first variant, and again calculate all characteristics of

*Pressure $p_{pr} = \dfrac{f\Delta}{1 - \Delta/\delta}$ is analogous to pressure $p_1 = \dfrac{f\Delta}{1 - \alpha\Delta}$ , but with respect to the initial free space $W_0\left(1 - \Delta/\delta\right)$ and not to the volume at the end of burning $W_0\left(1 - \alpha\Delta\right)$.

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of the mortar and the conditions of ignition; after comparing
the obtained data one should find that density of charge at which
the entire trajectory . will be the shortest (without an great
decrease in the coefficient $\eta_\omega$).After this, $p_{max}$ is converted into
$\pm$ 30-50 kg/cm$^2$; the characteristics of another series of variants
are calculated; and the largest given value for $p_{max}$ is determined.
The curves p, l and v, l are calculated for the final variant
selected.  The thickness of the powder $2e_1$ is determined only
for the final variant selected:

$$2e_1 = \frac{2u_1}{s}\sqrt{Bf\omega\varphi m}.$$

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